Morrell method for determining comminution circuit specific energy and assessing energy utilization efficiency of existing circuits

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1. FOREWORD

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Formed as part of the Canadian Institute of Mining, Metallurgy and Petroleum (CIM), GMG is supported by CIM and three other Partner Organizations: the Australasian Institute of Mining and Metallurgy (AusIMM), the Southern African Institute of Mining and Metals (SAIMM), and the Surface Mining Association for Research and Technology (SMART), as well as its Member Companies and participants.

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2. DEFINITIONS OF TERMS, SYMBOLS, AND ABBREVIATIONS

AG	Autogenous Grinding
F80	80% passing size of the circuit feed (µm)
Gpb	Grams (new minus closing screen aperture) per mill revolution (laboratory ball mill)
HPGR	High Pressure Grinding Roll
M_i	Generic term for hardness parameters M_{ia}, M_{ib}, M_{ic}, and M_{ih}
M_{ia}	Coarse ore (> 750 µm) work index in tumbling mill circuit(s) (kWh/t)
M_{ib}	Fine material (< 750 µm) work index in tumbling mill circuit(s) (kWh/t)
M_{ic}	Ore work index in crusher circuits (kWh/t)
M_{ih}	Ore work index in HPGR circuits (kWh/t)
P100	100% passing size or closing screen aperture (µm)
P80	80% passing size of the circuit product (µm)
SABC	Semi-Autogenous-Ball-Milling-Crushing
SAG	Semi-Autogenous Grinding
x_1–x_3	Generic terms for P80
W	Specific energy (work) input (kWh/t)
W_a	Specific energy to grind coarser particles in tumbling mills (kWh/t)
W_b	Specific energy to grind finer particles in tumbling mills (kWh/t)
W_c	Specific energy for conventional crushing (kWh/t)
W_h	Specific energy for HPGRs (kWh/t)
Wi	Bond Work Index
W_s	Specific energy correction for size distribution (kWh/t)
Wi_{BM}	Bond Ball Mill Test Work Index (kWh/t)

3. KEYWORDS

Autogenous Grinding (AG), Ball mill, Bond Work Index (Wi), Comminution circuit, High Pressure Grinding Rolls (HPGRs), Rod mill, Semi-Autogenous Grinding (SAG), Specific energy (W)

4. INTRODUCTION AND BACKGROUND

The Morrell method for predicting the specific energy consumption of conventional crushing, High Pressure Grinding Rolls (HPGRs), and tumbling mill equipment is well known and widely applied in the design of comminution circuits. The method is equally applicable to assessing the performance of operating comminution circuits. The Morrell method is described in full detail in Morrell (2004b, 2008, 2009); the GMG Morrell guideline is essentially a practical condensation of these works. The guideline reviews the data required for the analysis, including hardness characterization data generated from the SMC Test® (see Annex A) and the Bond Ball Mill Test Work Index (Wi_{BM}; GMG, 2016a), and the Morrell equations and their application (see Annex B). A worked example is provided in Annex C.

5. SCOPE

The Morrell method can be used to predict the specific energy of comminution circuits, where such circuits include combinations of any of the following equipment:
- Autogenous Grinding (AG) and Semi-Autogenous Grinding (SAG) mills
- Ball mills
- Rod mills
- Crushers
- HPGRs

Although the Morrell method can be used in comminution circuit design in greenfield projects, this document provides guidelines to use the method to assess the energy utilization efficiency of existing circuits.

6. OTHER USEFUL DOCUMENTS

The following referenced documents are indispensable for the application of this guideline:
- Global Mining Guidelines Group (GMG) (2016a). Determining the Bond Efficiency of industrial grinding circuits. Montreal, QC: Global Mining Guidelines Group.
- Global Mining Guidelines Group (GMG) (2016b). Methods to survey and sample grinding circuits for determining energy efficiency. Montreal, QC: Global Mining Guidelines Group.
- SMC Testing Pty Ltd. (2015). The SMC Test® is the most widely-used comminution test in the world for AG &

SAG Mills, HPGRs and Crushers. Retrieved on September 21, 2015, from http://www.smctesting.com/about

7. DATA REQUIREMENTS

7.1 From the Plant

The following data must obtained to assess the energy utilization efficiency of an existing circuit:
1. Identity of the relevant comminution equipment in the circuit
   - Typically this includes all crushers, HPGRs, and tumbling mills (AG/SAG, rod, and ball mills) involved in reducing the size of the primary crusher product to that of the final product (usually the cyclone overflow of the last stage of grinding prior to flotation/leaching).
2. Feed rate to the circuit (dry tonnes/h)
3. Power draw of the comminution equipment (kW)
   - In the case of mills, the power draw should be represented in terms of power at pinion for gear and pinion drives and at shell for gearless drives. For crushers, this should be the net power draw, that is, the gross (motor input) power draw less the no-load power.
4. Overall circuit specific energy: sum of the power draws of all comminution equipment divided by the circuit feed rate
5. 80% passing size (P80) of the primary crusher product (μm)
6. Product P80 of any intermediate crushing circuits treating primary crusher product (μm)
7. Product P80 of any intermediate HPGR circuit ahead of the tumbling mill stage(s) (μm)
8. Product P80 of the tumbling mill stage(s) (μm)
   - If there are multiple stages of grinding (e.g., SAG milling followed by ball milling), only the P80 of the product of the final milling stage is required.

In addition to the above data, a **representative** sample of the primary crusher product is required for subsequent laboratory hardness characterization.

7.2 From the Laboratory

The Morrell method uses hardness parameters obtained from the SMC Test® (SMC Testing Pty Ltd., 2015; Annex A) and the Bond Ball Mill Work Index Test (GMG, 2016a). The sampling and surveying guideline (GMG, 2016b) provides additional detail on how to collect the required data, and is critical to this analysis.

The following required parameters are standard outputs of the SMC Test®:

- M_{ia} describes grinding of coarser material (> 750 μm) in tumbling mill circuit(s).
- M_{ic} describes size reduction in crusher circuits.
- M_{ih} describes size reduction in HPGR circuits.

An additional required parameter (M_{ib}) is obtained from the data provided from a standard Wi_{BM} test. Note that the Wi_{BM} test should be carried out with a closing screen aperture that gives a final product P80 similar to that intended for the full-scale circuit.

- M_{ib} describes grinding of fine material (< 750 μm) in the tumbling mill circuit(s) and is calculated as follows (Morrell, 2008):

$$M_{ib} = \frac{18.18}{P100^{0.295} \times Gpb \times (P80^{f(P80)} - F80^{f(F80)})} \quad (1)$$

Where, P100 is the closing screen aperture (μm), Gpb is the net screen undersize product per revolution in the laboratory ball mill (g), P80 is 80% passing size of the product (μm), F80 is 80% passing size of the circuit feed (μm), f(P80) is

$$-(0.295 + \frac{P80}{1,000,000}), \text{ and f(F80) is } -(0.295 + \frac{F80}{1,000,000}).$$

8. MORRELL EQUATIONS

Given a circuit feed P80 and final product P80, plus the relevant hardness parameters, Morrell's equations can be used to predict the overall specific energy of most comminution circuit configurations. Full details of these equations are given in Annex B. However, they are all based on the same general energy-size reduction relationship represented by equation 2 (Morrell, 2004b).

$$W = M_i \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)}) \quad (2)$$

Where, W is the specific comminution energy (kWh/t), M_i refers to hardness parameters (i.e., work index related to the breakage property of an ore) from SMC and Wi_{BM} tests (kWh/t), x_2 is 80% passing size for the product or the P80 (μm), x_1 is the 80% passing size for the feed or the F80 (μm), and f(x_j) is

$$-(0.295 + \frac{x_j}{1,000,000}).$$

For tumbling mills, W relates to the power at the pinion or for gearless drives, the motor output. For HPGRs, W is the energy input to the rolls, whereas for conventional crushers, W relates to the specific energy as determined using the motor input power, less the no-load power.

The equations above were developed with aid of a database of 72 operating plants treating more than 110 ore types. The database covers all of the most popular circuit configurations. The equations reproduce the overall specific energies of these plants with a high degree of accuracy (1 standard deviation is 6.5% of the relative errors). The observed and predicted specific energies of all of these circuits are plotted in Figure 1.
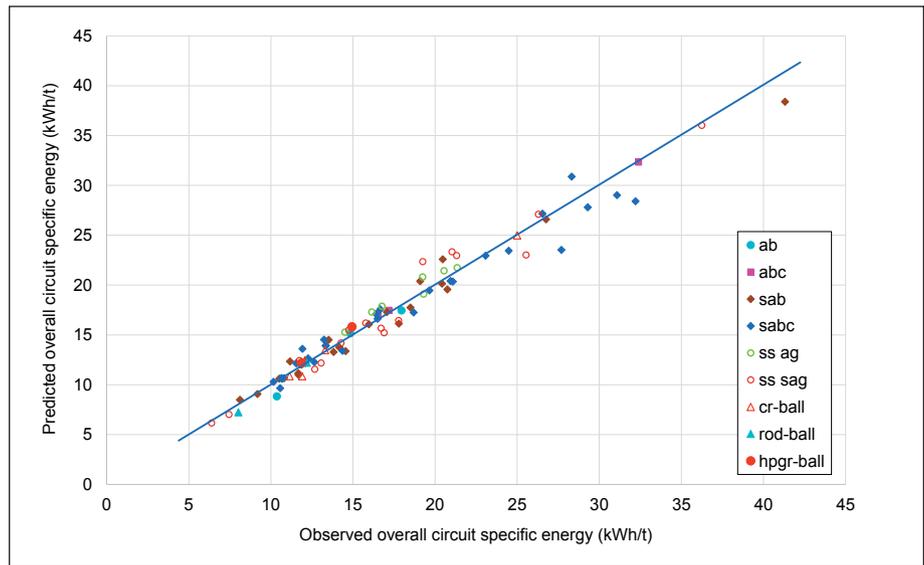
In Section 9, the equations are applied to three types of circuit to demonstrate their application to assessing the energy utilization of an existing plant. Annex C contains worked examples showing how these equations are used to predict the overall comminution circuit specific energy. The equations can also be found online on the SMC Testing website (http://www.smctesting.com/tools) in the form of a free "tool" that enables the user to obtain the overall circuit specific energy of most common circuits, given the relevant ore characterization values plus the F80 and P80 values.

Figure 1. Predicted vs. Observed Overall Circuit Specific Energy Using Morrell's Equations.
Note: a = autogenous; b = ball-milling; c or cr = crushing; g = grinding; sa = semi-autogenous; sabc = semi-autogenous mill with pebble crushing, followed by a ball mill; ss = single stage

9. USING THE MORRELL EQUATIONS

The data from the plant comprise a measured specific energy for the overall comminution circuit plus the F80 and P80 values. The SMC and Wi_{BM} tests supply the relevant hardness parameters of the feed ore. These hardness parameters are used in a series of equations (Annexes B and C) that predict the expected specific energy of the same circuit, assuming it is well run as judged by the standards of circuits in the database used to develop the equations (Figure 1).

Assume that the existing plant, which has a semi-autogenous mill with pebble crushing, followed by a ball mill (SABC circuit), was found to have an overall specific energy of 21.3 kWh/t. The measured feed size (F80) to the SAG mill was 100 mm and the measured ball mill cyclone overflow was 106 µm. SMC and Wi_{BM} tests on representative samples of the plant feed returned the following hardness parameters (in kWh/t):

$M_{ia} = 19.4$
$M_{ib} = 18.8$
$M_{ic} = 7.2$
$M_{ih} = 13.9$

Using these values in the relevant equations predicts that a well-run SABC circuit should consume on average 18.3 kWh/t to do the same duty as similar circuits from the database (see Annex C). The existing plant consumes 21.3 kWh/t, which is 16% more than predicted. Hence, the existing plant appears to be less efficient than expected. As mentioned in Section 8, for these equations, one standard deviation is 6.5% of the relative errors. The plant specific energy represents a difference of 2.5 standard deviations from the predicted value, which is highly significant (represents a situation that is expected to occur by chance with only a 0.5% probability). Therefore, a detailed investigation of plant operations would be warranted to determine the causes of the inefficiency and how to correct them.

The above analysis enables effective benchmarking of the performance of a given operating circuit against similar circuits elsewhere and indicates the extent to which energy utilization efficiency could be improved—in this case potentially by as much as 16%. However, application of the equations can be further extended by comparing the performance of a given circuit with different circuit configurations. For example, using the ore characteristics and F80 and P80 values above, the specific energy of a crushing/ball milling circuit or crushing/HPGR/ball milling circuit can be predicted and compared with the specific energy for a SABC circuit.

With reference to the worked examples in Annex C, the expected specific energy for a crushing/ball mill circuit is 16.4 kWh/t (see Section C.3.5), which equates to an energy savings of 10% compared to a well-run SABC circuit and

23% compared to our (not so well-run) existing circuit. If the HPGR circuit is considered, it would be expected to require 15.7 kWh/t, which would give even greater energy savings. However, caution needs to be exercised because only the direct comminution machine energy requirements are con- sidered in this analysis, and not ancillaries. Energy con- sumption tends to be higher for ancillaries in crushing/ball and HPGR/ball milling circuits than AG- and SAG-based cir- cuits, and hence the overall operating power differences tend to be slightly less than indicated.

10. REFERENCES AND RECOMMENDED READING

Buckingham, L., Dupont, J.-F., Stieger, J., Blain, B., & Brits, C. (2011). Improving energy efficiency in Barrick grinding circuits. *Proceedings of the 5th International Conference on Autoge- nous and Semiautogenous Grinding Technology (SAG 2011).* Montreal, QC: Canadian Institute of Mining, Metallurgy and Petroleum. 14 p.

Global Mining Guidelines Group (GMG) (2016a). Determining the Bond Efficiency of industrial grinding circuits. Montreal, QC: Global Mining Guidelines Group.

Global Mining Guidelines Group (GMG) (2016b). Methods to survey and sample grinding circuits for determining energy efficiency. Montreal, QC: Global Mining Guidelines Group.

JKTech (2011). JK Drop Weight Tester. Retrieved on September 22, 2015, from http://www.jktech.com.au/jk-drop-weight- tester

Morrell, S. (2004a). Predicting the specific energy of autogenous and semi-autogenous mills from small diameter drill core sam- ples. *Minerals Engineering, 17*(3), 447–451. http://dx.doi.org /10.1016/j.mineng.2003.10.019

Morrell, S. (2004b). An alternative energy-size relationship to that proposed by Bond for the design and optimisation of grinding circuits. *International Journal of Mineral Processing*, 74(1–4), 133–141. http://dx.doi.org/10.1016/j.minpro.2003.10.002

Morrell, S. (2006). Rock characterisation for high pressure grinding rolls circuit design. *Proceedings of the International Autogenous and Semi Autogenous Grinding Technology Con- ference, Vol. IV* (pp. 267–278).

Morrell, S. (2008). A method for predicting the specific energy requirement of comminution circuits and assessing their energy utilisation efficiency. *Minerals Engineering, 21*(3), 224– 233. http://dx.doi.org/10.1016/j.mineng.2007.10.001

Morrell, S. (2009). Predicting the overall specific energy requirement of crushing, high pressure grinding roll and tum- bling mill circuits. *Minerals Engineering, 22*(6), 544–549. http://dx.doi.org/10.1016/j.mineng.2009.01.005

SMC Testing Pty Ltd. (2015). The SMC Test® is the most widely- used comminution test in the world for AG & SAG Mills, HPGRs and Crushers. Retrieved on September 21, 2015, from http://www.smctesting.com/about

ANNEX A: SMC TEST®

A.1 General Description

The SMC Test® (SMC Testing Pty Ltd., 2015) uses either crushed rock pieces that are very closely sized from sieving ("crush and select" method; Figure A1) or particles that are cut to similar size from a drill core using a diamond saw ("cut-core" method; Figure A2). The former method is used when samples are sourced from feed to an existing plant. The latter method is used when drill core sample availability is limited. Almost any drill core size is suitable, even a core that has been quartered (slivered). The chosen particles are broken using a closely controlled range of impact energies with the JKTech Drop Weight Tester (JKTech, 2011). The raw data from breakage at these energies are processed by SMC Testing Pty Ltd. via JKTech and generate the ore hardness parameters, Drop Weight index, M_{ia}, M_{ic}, and M_{ih}, which are used in power-based equations, as well as the JKTech simulation parameters A, b, and ta. The specific gravity of the rock is also measured and reported.

Figure A1. Particles Selected for SMC Testing from Crushed Rock

A.2 Sample Quantity

The amount of sample that is required depends on the rock sample source (e.g., crushed rock pieces vs. drill core, particle/core size, and whole vs. halved vs. quartered core), as well as the size fraction chosen to do the SMC Test® and whether the sample is going to be prepared by crushing or cutting. These factors are best discussed with the metallurgical laboratory at the planning stage. However, in the majority of cases, 15–20 kg of sample is more than enough to conduct a single test. It should be remembered that the SMC Test® products can be re-used for Wi_{BM} testing, the SMC Test® effectively being used as a feed preparation step for the Wi_{BM} test. If the sample source is from an existing mine in operation, then sample quantity should not be a problem. In such cases, it is far better to be generous when selecting the sample and take more than is normally required. Good practice is to take at least twice the amount required and to retain half the material in case problems necessitate re-running the test.

Figure A2. Particles Selected for SMC Testing from Cutting a Drill Core

SMC Tests® can be carried out on three size fractions, depending on the nature and quantity of the feed sample:
–31.5 +26.5 mm
–22.4 +19.0 mm
–16.0 +13.2 mm

If material quantity and size that is available for testing is no object (e.g., when the sample comes from an existing operation), then the –31.5 +26.5 mm fraction is recommended.

Note that **no calibration** is required to generate the M_{ia}, M_{ih}, and M_{ic} parameters from a SMC Test®: they are produced as a standard output from the SMC Test®. The question of calibration only arises when the JK simulation parameters A and b are required from the test. Even in these cases, most often calibration is not required.

ANNEX B: MORRELL EQUATIONS

B.1 General Description

The Morrell approach divides comminution equipment into three categories:

1. Tumbling mills (e.g., AG, SAG, rod, and ball mills)
2. Conventional reciprocating crushers (e.g., jaw, gyratory, and cone)
3. HPGRs

Tumbling mills are described using two work indices (M_{ia} and M_{ib}), whereas crushers and HPGRs each have one work index (M_{ic} and M_{ih}, respectively) (Morrell, 2008, 2009).

- M_{ia} describes grinding of coarser material (P80 > 750 μm up to the P80 of the product of the last stage of crushing or HPGR size reduction prior to grinding) in tumbling mill circuit(s).
- M_{ib} describes grinding of fine material (P80 < 750 μm down to P80 sizes typically reached by conventional ball milling, or approximately 45 μm) in tumbling mill circuit(s).
- M_{ic} describes size reduction in crusher circuits.
- M_{ih} describes size reduction in HPGR circuits.

M_{ia} values are provided as a standard output from a SMC Test® (Morrell, 2004a), whereas M_{ib} values can be determined using the data generated by a conventional Wi_{BM} test (M_{ib} is NOT the Wi_{BM}). M_{ic} and M_{ih} values are also provided as a standard output from a SMC Test® (Morrell, 2009).

For tumbling mills, M_{ia} and M_{ib} relate to coarse and fine ore properties, respectively. There is an additional efficiency factor that represents the influence of a pebble crusher in AG/SAG mill circuits. The choice of 750 μm as the division between "coarse" and "fine" particle sizes was determined during the development of the technique and was found to give the best overall results across the range of plants in the SMCC Pty Ltd. database. Implicit in the approach is that distributions are parallel and linear in log-log space (see Section B.2.4). See equation 2 for the general size reduction equation from Morrell (2004b).

B.2 Specific Energy Determination for Comminution Circuits

The total specific energy (W_T in kWh/t) to reduce in size the primary crusher product to the final product is given by:

$$W_T = W_a + W_b + W_c + W_h + W_s \qquad (B1)$$

Where, W_a is the specific energy to grind coarser particles in tumbling mills, W_b is the specific energy to grind finer particles in tumbling mills, W_c is the specific energy for conventional crushing, W_h is the specific energy for HPGRs, and W_s

is the specific energy correction for size distribution (all in kWh/t).

Clearly only the W values associated with the relevant equipment in the circuit being studied are included in equation B1.

B.2.1 Tumbling Mills

To determine the specific energy to grind coarse particles (> 750 μm) in tumbling mills (W_a), equation 2 is written as:

$$W_a = K_1 \times M_{ia} \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)}) \qquad (B2)$$

Where, $K_1 = 1.0$ for all circuits that do not contain a recycle pebble crusher, $K_1 = 0.95$ where circuits have a pebble crusher, M_{ia} is the coarse ore work index (kWh/t), x_2 is set to 750 μm, x_1 is the P80 of the circuit feed, that is, the product of the last stage of crushing before grinding (also the feed for grinding or the F80; μm), $f(x_2)$ is

$$-(0.295 + \frac{750}{1,000,000}), \text{ and } f(x_1) \text{ is} -(0.295 + \frac{x_1}{1,000,000}).$$

To determine the specific energy to grind fine particles (< 750 μm) in tumbling mills (W_b), equation 2 is written as:

$$W_b = M_{ib} \times 4 \times (x_3^{f(x_3)} - x_2^{f(x_2)}) \qquad (B3)$$

Where, M_{ib} is the fine material work index (kWh/t), x_3 is the P80 of the final grind (μm), x_2 is set to 750 μm, $f(x_3)$ is

$$-(0.295 + \frac{x_3}{1,000,000}), \text{ and } f(x_2) \text{ is} -(0.295 + \frac{750}{1,000,000}).$$

The M_{ib} is calculated from data from the standard Wi_{BM} test using equation 1 (Morrell, 2008). Note that the Wi_{BM} test should be carried out with a closing screen mesh size that gives a final product P80 similar to that intended for the full-scale circuit.

B.2.2 Conventional Crushers

To determine the specific energy for conventional crushing (W_c), equation 2 is written as:

$$W_c = S_c \times K_2 \times M_{ic} \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)}) \qquad (B4)$$

Where, S_c is the coarse ore hardness parameter used in primary and secondary crushing situations (see equation B5), $K_2 = 1.0$ for crushers operating in closed circuit with a classifying screen, $K_2 = 1.19$ for crushers operating in open circuit (e.g., pebble crusher in an AG/SAG circuit), M_{ic} is the crushing ore work index provided directly by the SMC Test®

(kWh/t), x_2 is the P80 of the circuit product (µm), x_1 is the P80 of the circuit feed (µm), $f(x_2)$ is

$$-\left(0.295 + \frac{x_2}{1,000,000}\right), \text{ and } f(x_1) \text{ is } -\left(0.295 + \frac{x_1}{1,000,000}\right).$$

The parameter S_c accounts for the decrease in ore hardness that becomes significant in relatively coarse crushing applications such as primary and secondary cone/gyratory circuits. In tertiary and pebble crushing circuits, it is normally not necessary and takes the value of unity. In full-scale HPGR circuits—where feed sizes tend to be higher than those used in laboratory and pilot scale machines—the parameter has also been found to improve predictive accuracy. The parameter S is defined by the general equation B5:

$$S = K_s(x_1 \times x_2)^{-0.2} \qquad (B5)$$

Where, K_s is a machine-specific constant related to whether it is a conventional crushing circuit or an HPGR circuit (see Section B.2.3), x_2 is the P80 of the circuit product (µm) and x_1 is the P80 of the circuit feed (µm). In the case of conventional crushing circuits, K_s is set to 55 and the S parameter is referred to as S_c.

To determine whether S_c should be applied in a given crushing circuit, when the value of S_c is > 1, it should not be applied (i.e., $S_c = 1$) and when the value of S_c is < 1, it should be applied.

B.2.3 HPGRs

To determine the specific energy for HPGRs (W_h), equation 2 is written as:

$$W_h = S_h \times K_3 \times M_{ih} \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)}) \qquad (B6)$$

Where, S_h is the coarse ore hardness parameter used in HPGRs (substitute S with S_h in equation B5, with K_s set to 35), $K_3 = 1.0$ for HPGRs operating in closed circuit with a classifying screen, $K_3 = 1.19$ for HPGRs operating in open circuit, M_{ih} is the ore work index provided directly by the SMC Test® (kWh/t), x_2 is the P80 of the circuit product (µm), x_1 is the P80 of the circuit feed (µm), $f(x_2)$ is

$$-\left(0.295 + \frac{x_2}{1,000,000}\right), \text{ and } f(x_1) \text{ is } -\left(0.295 + \frac{x_1}{1,000,000}\right).$$

To determine whether S_h should be applied in a given HPGR circuit, when the value of S_h is > 1, it should not be applied (i.e., $S_h = 1$) and when the value of S_h is < 1, it should be applied.

B.2.4 Specific Energy Correction for Size Distribution (W_s)

The approach described in this guideline assumes that the feed and product size distributions are parallel and linear in log-log space. If they are not, corrections are required. These corrections are most likely to be necessary in circuits where closed circuit secondary/tertiary crushing is followed by ball milling, because such crushing circuits tend to produce a product size distribution that is relatively steep compared to the ball mill circuit cyclone overflow. This is illustrated in Figure B1, which shows measured distributions from an open and closed crusher circuit, as well as a ball mill cyclone overflow. The closed circuit crusher distribution is steeper than the open circuit crusher distribution and ball mill cyclone overflow. Also the open circuit distribution more closely follows the gradient of the cyclone overflow.

If a ball mill circuit was fed two distributions, each with the same P80, but with the open and closed circuit gradients in Figure B1, the closed circuit distribution would
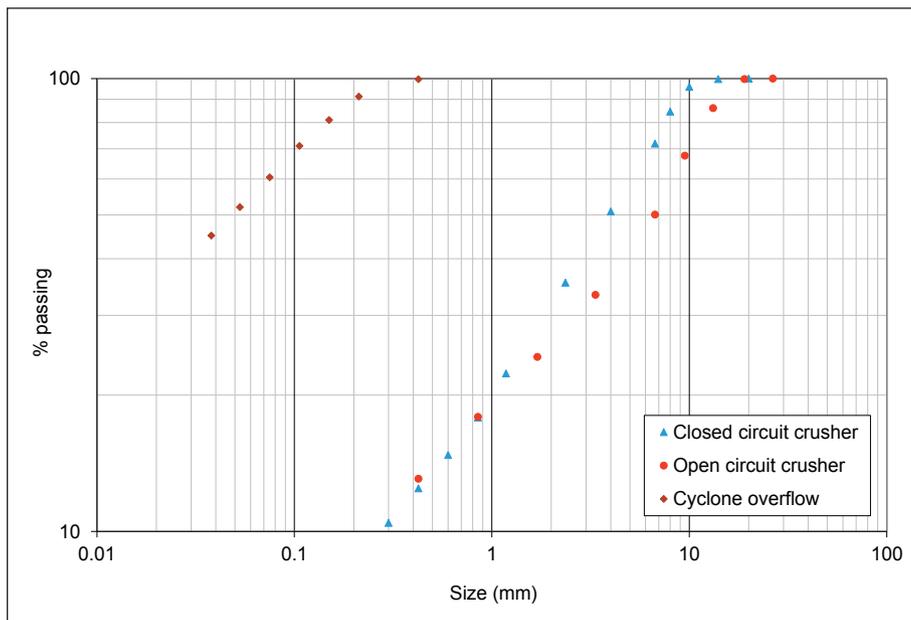

Figure B1. Examples of Open and Closed Circuit Crushing Size Distributions Compared with a Typical Ball Mill Cyclone Overflow Distribution

require more energy to grind to the final P80. How much more energy is difficult to determine. However, it has been assumed that the additional specific energy for ball milling is the same as the difference in specific energy between open and closed crushing to reach the nominated ball mill feed size. The crusher is assumed to provide this energy. However, the ball mill has to supply this energy and it has a higher work index than the crusher (i.e., the ball mill is less energy efficient than a crusher and has to input more energy to do the same amount of size reduction). Hence from equation B4, to crush to the ball mill circuit feed size (x_2) in open circuit requires specific energy equivalent to:

$$W_c = 1 \times 1.19 \times M_{ic} \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)})$$

And from equation B4, to crush to the ball mill circuit feed size (x_2) in closed circuit requires specific energy equivalent to:

$$W_c = 1 \times 1 \times M_{ic} \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)})$$

The energy difference between the two equations above has to be provided by the milling circuit, allowing for the fact that the ball mill—with its lower energy efficiency—has to provide the energy, not the crusher. This energy is the W_s (equation B1) and for the above example is represented by:

$$W_s = 0.19 \times M_{ia} \times 4 \times (x_2^{f(x_2)} - x_1^{f(x_1)})$$

Note that M_{ic} from the previous two equations has been replaced with M_{ia}, the coarse particle tumbling mill grinding work index. Also, S_c was set to unity because typically a tertiary crushing stage feeds the ball mill; S_c takes the value of 1 under these circumstances.

In AG/SAG-based circuits, W_s appears to be unnecessary. Product distributions in primary crusher feeds often have the shape shown in Figure B2, which has a very similar gradient to typical ball mill cyclone overflows.

A similar situation appears to apply with HPGR product size distributions (Figure B3). Interestingly the SMCC Pty Ltd. data show that for HPGRs, closed circuit operation appears to require a lower specific energy to reach the same P80 as open circuit operation, even though the distributions for open and closed circuit appear to have almost identical gradients. Closer examination of the distributions shows that in closed circuit, the final product tends to have slightly less very fine material, which could account for the different
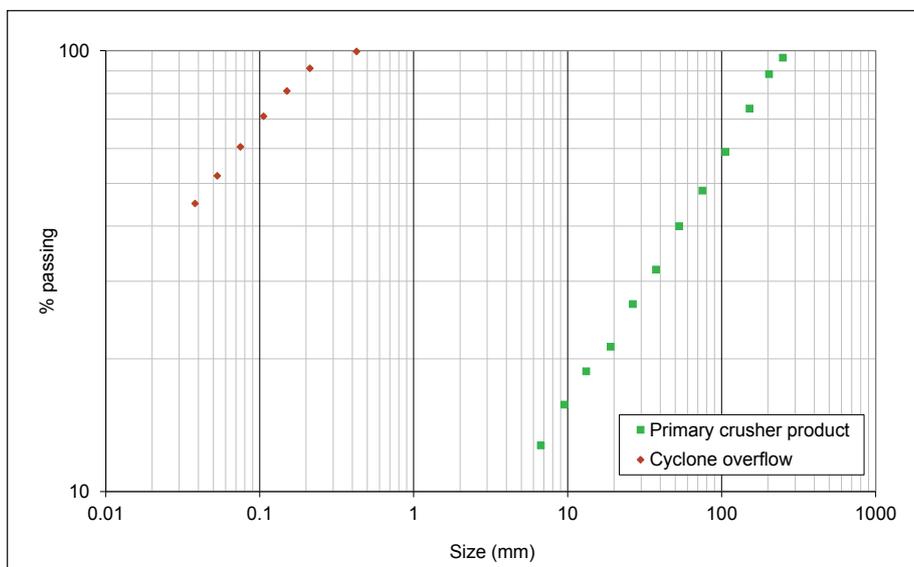

Figure B2. Example of a Typical Primary Crusher (Open Circuit) Product Size Distribution Compared with a Typical Ball Mill Cyclone Overflow Size Distribution
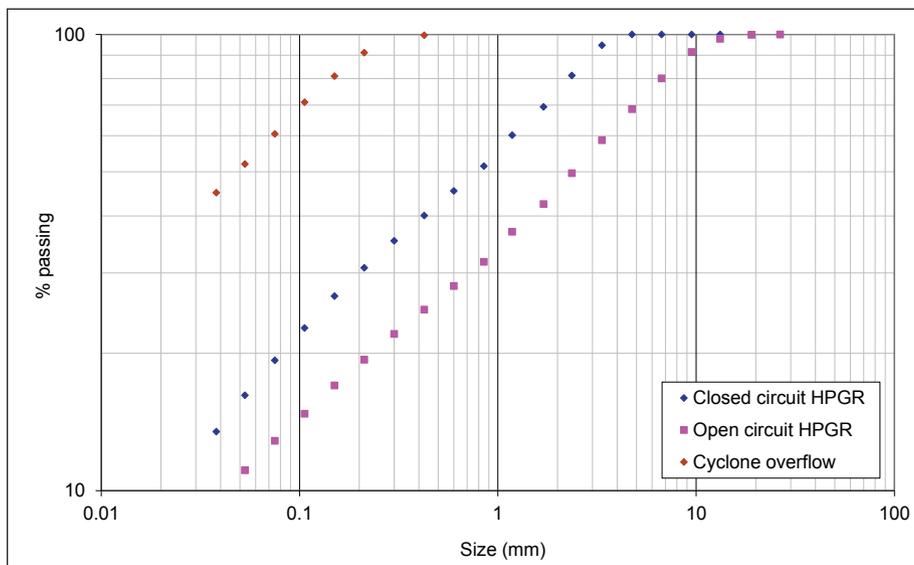

Figure B3. Examples of Open and Closed Circuit HPGR Size Distributions Compared with a Typical Ball Mill Cyclone Overflow Size Distribution

energy requirements between the two modes of operation. It is also possible that recycled material in closed circuit is inherently weaker than new feed, because it has already passed through the HPGR and could have sustained micro-cracking. A reduction in the Wi_{BM} as measured by testing HPGR products (compared it to the Wi_{BM} of HPGR feed) has been detected in many cases in the laboratory (see Section B.2.5), and hence there is no reason to expect the same phenomenon would not affect the recycled HPGR screen oversize.

It follows from the above arguments that in HPGR circuits, which are typically fed with material from closed circuit secondary crushers, a similar feed size distribution correction should be applied. However, as the secondary crushing circuit uses little energy relative to the rest of the circuit (because it crushes to a relatively coarse size), the magnitude of size distribution correction is very small indeed— much smaller than the error associated with the technique—and hence may be omitted in calculations.
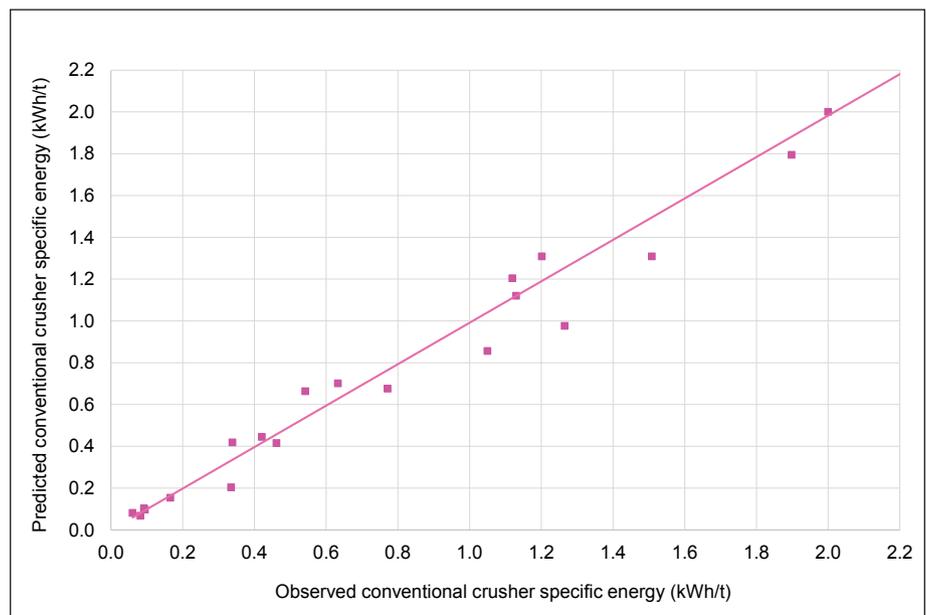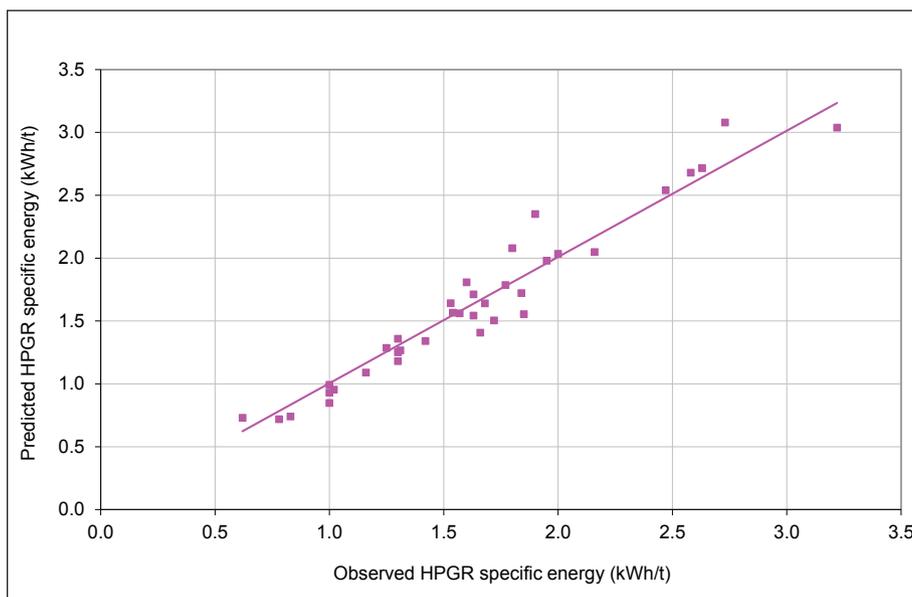
be reduced by an amount that the user thinks is appropriate. Until more data become available from full-scale HPGR/ball mill circuits, it is suggested that—in the absence of Wi_{BM} data on HPGR products—the M_{ib} results from HPGR feed material are reduced by no more than 5% to allow for the effects of micro-cracking.
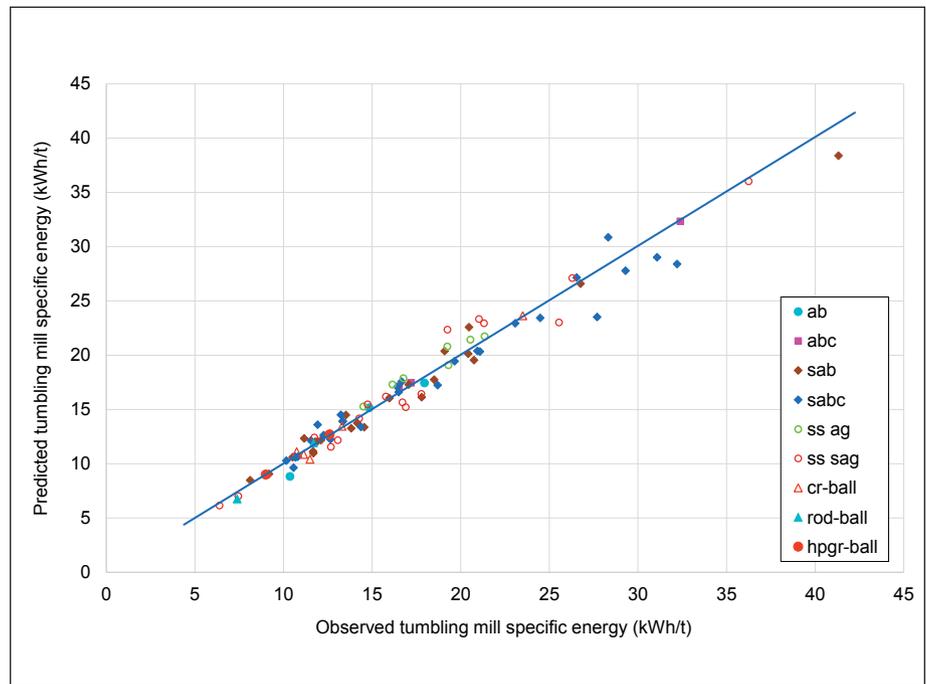
Figure B4. Predicted vs. Observed Tumbling Mill Circuit Specific Energy. Note: a = autogenous; b = ball-milling; c or cr = crushing; g = grinding; sa = semi-autogenous; sabc = semi-autogenous mill with pebble crushing, followed by a ball mill; ss = single stage

B.2.5 Weakening of HPGR Products

Various researchers have reported experimental laboratory results showing that the Wi_{BM} is lower for HPGR products than feed. The magnitude of this reduction varies with both the material type and the pressing force used, but is typically < 10%. In the approach here, no allowance has been made for such weakening. However, if HPGR products are available to conduct Wi_{BM} tests, then M_{ib} values obtained from such tests can be used in equation B3. Alternatively, the M_{ib} values from Wi_{BM} tests on HPGR feed material can

Figure B5. Predicted vs. Observed Conventional Crusher Specific Energy

Figure B6. Predicted vs. Observed HPGR Specific Energy

B.3 Validation

B.3.1 Tumbling Mill Circuits

The approach described in Section B.2 was applied to applied to 65 industrial datasets (Figure B4). In all cases, the specific energy relates to the tumbling mills contributing to size reduction from the product of the final stage of crushing to the final grind. Data are presented in terms of equivalent specific energy at the pinion. It was assumed that power at the pinion was 93.5% of the measured gross (motor input) power, this value being typical of what is normally accepted to represent losses across the motor and gearbox. For gearless drives (so-called wrap-around motors) a value of 97% was used.

B.3.2 Conventional Crushers

Validation of equation 2 used 12 crushing circuits (25 datasets), including secondary, tertiary, and pebble crushers in AG/SAG circuits. Observed vs. predicted specific energies are given in Figure B5. The observed specific energies were calculated from the crusher throughput and the net power draw of the crusher as defined by:

$$\text{Net power} = \text{Motor input power} - \text{No-load power} \quad (B7)$$

No-load power tends to be relatively high in conventional crushers and hence net power is significantly lower than the motor input power. Examination of the 25 crusher datasets showed the motor input power was on average 35% higher than the net power.

B.3.3 HPGRs

Validation of equation 2 for HPGRs used data from 19 circuits (36 datasets), including laboratory, pilot, and industrial-scale equipment. Observed vs. predicted specific energies are given in Figure B6. The data relate to HPGRs operating with specific grinding forces typically in the range of 2.5–3.5 N/mm^2. The observed specific energies relate to power delivered by the roll drive shafts. Motor input power for full-scale machines is expected to be 8–10% higher.

ANNEX C: WORKED EXAMPLES

The goal is to estimate the overall specific grinding energy to reduce a primary crusher product with a P80 of 100 mm to a final product P80 of 106 μm. SMC and Wi_{BM} tests were carried out on a representative ore sample (Table C1).

Three circuits are evaluated: SABC, HPGR/ball mill, and conventional crushing/ball mill.

Table C1. Values used for specific grinding energy calculations

Parameter (kWh/t)	Value	Test
M_{ia}	19.4	SMC
M_{ib}	18.8	Wi_{BM}
M_{ic}	7.2	SMC
M_{ih}	13.9	SMC

C.1 SABC Circuit

C.1.1 Coarse Particle Tumbling Mill Specific Energy

Using equation B2 from Annex B:

$$W_a = 0.95 \times 19.4 \times 4 \times \left(750^{-(0.295 + \frac{750}{1,000,000})} - 100,000^{-(0.295 + \frac{100,000}{1,000,000})}\right)$$
$$= 9.6 \text{ kWh/t}$$

C.1.2 Fine Particle Tumbling Mill Specific Energy

Using equation B3 from Annex B:

$$W_b = 18.8 \times 4 \times \left(106^{-(0.295 + \frac{106}{1,000,000})} - 750^{-(0.295 + \frac{750}{1,000,000})}\right)$$
$$= 8.4 \text{ kWh/t}$$

C.1.3 Pebble Crusher Specific Energy

In this circuit, the pebble crusher feed P80 is assumed to be 52.5 mm. As a rule of thumb, this value can be estimated as 0.75 of the nominal pebble port aperture (in this case the pebble port aperture is 70 mm). The pebble crusher is set to give a product P80 of 12 mm. The pebble crusher feed rate is expected to be 25% of new feed rate.

Using equation B4 from Annex B:

$$W_c = 1.19 \times 7.2 \times 4 \times \left(12,000^{-(0.295 + \frac{12,000}{1,000,000})} - 52,500^{-(0.295 + \frac{52,500}{1,000,000})}\right)$$
$$= 1.13 \text{ kWh/t}$$

The product of this calculation is 1.13 kWh/t when expressed in terms of the crusher feed rate. It is 0.3 kWh/t (1.13 × 0.25) when expressed in terms of the SABC circuit new feed rate.

C.1.4 Total Net Comminution Specific Energy

Using equation B1 from Annex B:

$$W_T = 9.6 + 8.4 + 0.3 = 18.3 \text{ kWh/t}$$

C.2 HPGR/Ball Mill Circuit

In this circuit, primary crusher product is reduced to a HPGR circuit feed P80 of 35 mm by closed circuit secondary crushing. The HPGR is also in closed circuit and reduces the 35 mm feed to a circuit product P80 of 4 mm. This product is then fed to a closed circuit ball mill, which takes the grind down to a P80 of 106 μm.

C.2.1 Secondary Crushing Specific Energy

Combining equations B4 and B5 from Annex B:

$$W_c = 55 \times (35,000 \times 100,000)^{-0.2} \times 1 \times 7.2 \times 4 \times$$
$$(35,000^{-(0.295 + \frac{35,000}{1,000,000})} - 100,000^{-(0.295 + \frac{100,000}{1,000,000})})$$
$$= 0.4 \text{ kWh/t}$$

C.2.2 HPGR Specific Energy

Combining equations B5 and B6 from Annex B:

$$W_h = 35 \times (4,000 \times 35,000)^{-0.2} \times 1 \times 13.9 \times 4 \times$$
$$(4,000^{-(0.295 + \frac{4,000}{1,000,000})} - 35,000^{-(0.295 + \frac{35,000}{1,000,000})})$$
$$= 2.4 \text{ kWh/t}$$

C.2.3 Coarse Particle Tumbling Mill Specific Energy

Using equation B2 from Annex B:

$$W_a = 1 \times 19.4 \times 4 \times$$
$$(750^{-(0.295 + \frac{750}{1,000,000})} - 4,000^{-(0.295 + \frac{4,000}{1,000,000})})$$
$$= 4.5 \text{ kWh/t}$$

C.2.4 Fine Particle Tumbling Mill Specific Energy

Using equation B3 from Annex B:

$$W_b = 18.8 \times 4 \times$$
$$(160^{-(0.295 + \frac{106}{1,000,000})} - 750^{-(0.295 + \frac{750}{1,000,000})})$$
$$= 8.4 \text{ kWh/t}$$

C.2.5 Total Net Comminution Specific Energy

Using equation B1 from Annex B:

$$W_T = 0.4 + 2.4 + 4.5 + 8.4 = 15.7 \text{ kWh/t}$$

C.3 Conventional Crushing/Ball Mill Circuit

In this circuit, primary crusher product is initially reduced in size to a P80 of 35 µm in an open circuit secondary crusher. This material is then reduced in size to a P80 of 6.5 µm via a closed tertiary/quaternary crushing circuit. This product is then fed to a closed circuit ball mill, which grinds to a P80 of 106 µm.

C.3.1 Secondary Crushing Specific Energy

Combining equations B4 and B5 from Annex B:

$$W_c = 55 \times (35{,}000 \times 100{,}000)^{-0.2} \times 1.19 \times 7.2 \times 4 \times$$

$$(35{,}000^{-(0.295 + \frac{35{,}000}{1{,}000{,}000})} - 100{,}000^{-(0.295 + \frac{100{,}000}{1{,}000{,}000})})$$

$$= 0.5 \text{ kWh/t}$$

C.3.2 Tertiary/Quaternary Crushing Specific Energy

Combining equations B4 and B5 from Annex B:

$$W_c = 1 \times 1 \times 7.21 \times 4 \times$$

$$(6{,}500^{-(0.295 + \frac{6{,}500}{1{,}000{,}000})} - 35{,}000^{-(0.295 + \frac{35{,}000}{1{,}00{,}0{,}000})})$$

$$= 1.1 \text{ kWh/t}$$

Note that in this case, $S_c = 551 \times (6{,}500 \times 35{,}000)^{-0.2}$ =1.17. Because it is greater than unity, S_c does not apply and is set to 1.

C.3.3 Coarse Particle Tumbling Mill Specific Energy

Combining equations B4 and B5 from Annex B:

$$W_a = 1 \times 19.4 \times 4 \times$$

$$(750^{-(0.295 + \frac{750}{1{,}000{,}000})} - 6{,}500^{-(0.295 + \frac{6{,}500}{1{,}00{,}0{,}000})})$$

$$= 5.5 \text{ kWh/t}$$

C.3.4 Fine Particle Tumbling Mill Specific Energy

Using equation B3 from Annex B:

$$W_b = 18.8 \times 4 \times$$

$$(106^{-(0.295 + \frac{106}{1{,}000{,}000})} - 750^{-(0.295 + \frac{750}{1{,}000{,}000})})$$

$$= 8.4 \text{ kWh/t}$$

C.3.5 Size Distribution Correction

$$W_s = 0.19 \times 19.4 \times 4 \times$$

$$(6{,}500^{-(0.295 + \frac{6{,}500}{1{,}000{,}000})} - 1{,}000{,}000^{-(0.295 + \frac{100{,}000}{1{,}000{,}000})})$$

$$= 0.9 \text{ kWh/t}$$

C.3.6 Total Net Comminution Specific Energy

Using equation B1 from Annex B:

$$W_T = 0.5 + 1.1 + 5.5 + 8.4 + 0.9 = 16.4 \text{ kWh/t}$$